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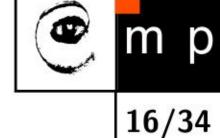




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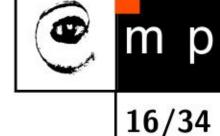
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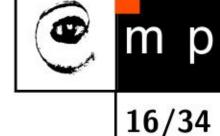
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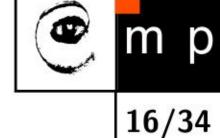


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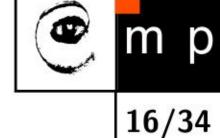




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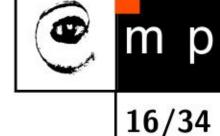
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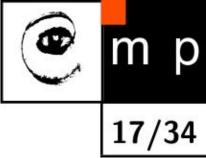
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These tribes correspond to set-representable  $\sigma$ -complete MV-algebras



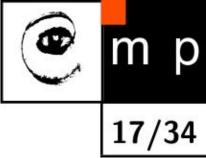
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**Theorem:** [Butnariu, Klement] All elements of T are B-measurable. Each measure is **regular** and it is of the form

$$\mu(A) = \int A \, d\nu$$

where  $\nu = \mu \upharpoonright \mathcal{B}$  is a (classical) measure on  $\mathcal{B}$ .



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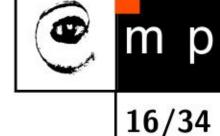
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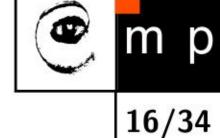


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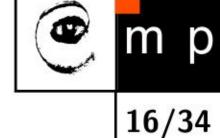




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**Always:** Crisp elements of  $\mathcal{T}$ , i.e.,  $\mathcal{T} \cap \{0,1\}^X$ , determine a  $\sigma$ -algebra  $\mathcal{B}$ 

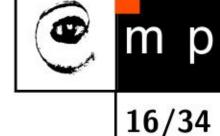
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$$(A \cap B)(x) = T(A(x), B(x)), \qquad (A \cup B)(x) = S(A(x), B(x))$$



**Example:** Let  $\mathcal{B}$  be a  $\sigma$ -algebra of subsets of X,

 $\mathcal{T}$  be the corresponding collection of characteristic functions (indicators):

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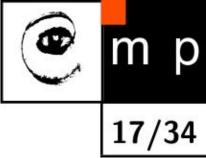
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These tribes correspond to set-representable  $\sigma$ -complete MV-algebras



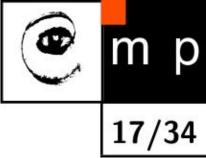
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**Theorem:** [Butnariu, Klement] All elements of T are B-measurable. Each measure is **regular** and it is of the form

$$\mu(A) = \int A \, d\nu$$

where  $\nu = \mu \upharpoonright \mathcal{B}$  is a (classical) measure on  $\mathcal{B}$ .



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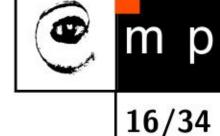
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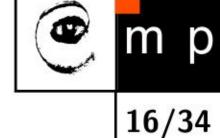


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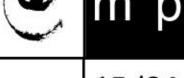


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# Basic notions of fuzzy measure theory



classical measure theory	fuzzy measure theory
$\sigma$ -algebra $\mathcal{T} \subseteq 2^X$	tribe $(\mathcal{T},T)$ , where $\mathcal{T}\subseteq [0,1]^X$
$\emptyset \in \mathcal{T}$	$0 \in \mathcal{T}$
$A \in \mathcal{T} \Rightarrow A' = X \setminus A \in \mathcal{T}$	$A \in \mathcal{T} \Rightarrow A' = 1 - A \in \mathcal{T}$
$A, B \in \mathcal{T} \Rightarrow A \cap B \in \mathcal{T}$	$A, B \in \mathcal{T} \Rightarrow A \cap B \in \mathcal{T}^*$
$(A_n)_{n\in\mathbb{N}}\subseteq\mathcal{T}, A_n\nearrow A\Rightarrow A\in\mathcal{T}$	$(A_n)_{n\in\mathbb{N}}\subseteq\mathcal{T}, A_n\nearrow A\Rightarrow A\in\mathcal{T}$
measure $\mu$ : $\mathcal{T} \to [0, \infty[$	regular measure $\mu$ : $\mathcal{T} \to [0, \infty[$
$\mu(\emptyset) = 0$	$\mu(0) = 0$
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